

# B.Sc. B.Ed SEM-I Examination: 2019

## Course-CC1

### Subject: Elementary Algebra and Calculus

Time: 2 Hours

F.M. 50

Answer any *ten* questions

(5 × 10 = 50)

1. Prove that  $\sin\{i \log \frac{p-iq}{p+iq}\} = \frac{2pq}{p^2+q^2}$  where  $p, q$  are real and  $(p, q) \neq (0,0)$ .
2. Solve the equation  $x^3 - 3x + 1 = 0$  by Cardan's method.
3. If  $a, b, c$  are positive real numbers then prove that  $(a^2b + b^2c + c^2a)(ab^2 + bc^2 + ca^2) \geq 9a^2b^2c^2$ .
4. Determine the value of 'c' and 'd' so that the system of equation  $x + 2y + z = 1$ ;  $3x + y + 2z = d$ ;  $cx - y + 4z = d^2$  has (i) a unique solution, (ii) no solution, (iii) many solutions in the field of real numbers.
5. Determine the rank of the matrix  $A =$

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{pmatrix}$$

6. Find the inverse of  $A$ , where  $A =$

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{pmatrix}$$

7. Show that, the function  $g$  defined by  $g(x) = x^2 \sin \frac{1}{x}$  when  $x \neq 0$  and  $g(0) = 0$  is differentiable everywhere but the derived function  $g'$  is not continuous at  $x = 0$ .
8. If  $y = e^{ax} \cos bx$ , show that  $y'' - 2ay' + (a^2 + b^2)y = 0$
9. Use L Hospital's rule to prove the following limit.

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$$

10. Find the reduction formula for  $\int \sin^m x \cos^n x dx$  where  $m$  and  $n$  being positive integer, greater than 1.
11. The part of the parabola  $y^2 = 4ax$  bounded by the latus rectum revolves about the tangent at the vertex. Find the volume and the area of the curved surface of the reel generated.
12. Show that

$$\int_0^{\frac{\pi}{2}} \cos^m x dx = \int_0^{\frac{\pi}{2}} \sin^m x dx = \frac{\sqrt{\pi}}{2} \frac{\Gamma(\frac{m+1}{2})}{\Gamma(\frac{m+2}{2})} \text{ Where, } m > -1$$